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06MAT31

Third Semester B.E. Degree Examination, Dec.2013/Jan.2014
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Obtain the Fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $(0,2\pi)$ and deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....=\frac{\pi^2}{6}$ **(07 Marks)**

b. Compute the constant term and the first two harmonics in the Fourier series of f(x) given by the following table: **(07 Marks)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

c. Expand the function f(x) defined by,

$$f(x)=\begin{cases}\frac{1}{4}-x, & \text{for } 0<x<\frac{1}{2}\\ x-\frac{3}{4}, & \text{for } \frac{1}{2}<x<1\end{cases}$$

in a half range sine series. **(06 Marks)**

2 a. Find the Fourier transform of, $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|>1\end{cases}$

Hence evaluate $\int_0^\infty \frac{x\cos x+\sin x}{x^3}.\cos\left(\frac{x}{2}\right)dx$ **(07 Marks)**

b. Find the Fourier cosine transform of e^{-ax} and xe^{-ax} where a>0 deduce that, $\int_0^\infty \frac{\cos mx dx}{x^2+a^2}=\frac{\pi}{2a}e^{-am}$ **(07 Marks)**

c. Find the finite Fourier sine transform of the function, $f(x)=\begin{cases}-x, & 0<x<a\\ \pi-x, & a<x<\pi\end{cases}$ where a is a constant over the interval $(0,\pi)$. **(06 Marks)**

3 a. Find the partial differential equation arising from the equation $\phi(x+y+zxy+z^2)=0$, where φ is an arbitrary function. **(07 Marks)**

b. Solve : $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ **(07 Marks)**

c. Solve : $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0$ by the method of separation of variables. **(06 Marks)**

4 a. Derive the one-dimensional heat equation. **(07 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, subject to the boundary conditions, $u(0,t) = 0$, $u(1,t) = 0$, $t \geq 0$ and the initial conditions $u(x, 0) = \sin\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$, $0 < x < 1$ by taking $h = \frac{1}{4}$ and $K = \frac{1}{5}$. Find the second level solution in the time scale. **(07 Marks)**

c. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for the following sequence mesh with boundary values as shown. Carry out two iterations. **(06 Marks)**

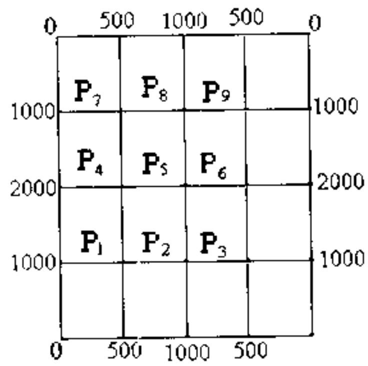

Fig. Q4 (c)

PART – B

5 a. Using Newton-Raphson method, find the root of $x \log_{10} x = 1.2$ near 2.5. Carryout three iterations. **(07 Marks)**

b. Apply Gauss-Seidel iterative method to solve,

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

Carryout four iterations, taking the initial approximation to the solution as $(1, 0, 3)$. **(07 Marks)**

c. Use the power method to find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking the initial approximation to the corresponding eigen vector as $[1, 1, 1]^T$. Perform 5 iterations. **(06 Marks)**

6 a. Using Newton's divided difference formula find f(8) and f(15) from the following data: **(07 Marks)**

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

b. A rod is rotating in a plane the following gives the angle θ radians through which the rod has turned for various values of the time t second.

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and angular acceleration of the rod when t = 0.6 second. Employ Newton's Forward-interpolation formula. **(07 Marks)**

6 c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce that the value of $\log_e 2$. **(06 Marks)**

7 a. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

b. Find the extremal of the functional, $I = \int_0^{\pi/2} \left(y^2 - y'^2 - 2y \sin x\right) dx$ under the end conditions $y(0) = y(\pi/2) = 0$ **(07 Marks)**

c. State and prove the Brachistochrone problem. **(06 Marks)**

8 a. Find the z-transforms of (i) $\cos n\theta$ (ii) $n \cos n\theta$ **(07 Marks)**

b. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)4}$, find u_0, u_1, u_2, u_3. **(07 Marks)**

c. Solve $u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$ by using the z-transform. **(06 Marks)**

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